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RECURRENCE FORMULAS IN THE PROBABILITY THEORY

People have been watching in statistical observations since ancient times, but also in our time statistics become an integral part of the most diverse spheres of human activity. Currently, it is impossible to imagine modern management of any industry, media work, politics, advertising, many other areas of our existence without the processing and analysis of statistical materials. For example, during the pandemic COVID-19 creating vaccines against the coronavirus strain necessarily involved statistical processing of the results of their tests and further observations. The conduct of hostilities also involves the collection of information and its processing in order to establish certain areas of improvement in both military tactics and weapons.

Also the same time, it is important not only to collect qualitative statistical data, and interpret them correctly and draw right conclusions, and this is quite not such a simple matter.

Let's give a visual example. We will conduct a test that consists of a

question on specific knowledge and involves choosing an answer or «yes», or «no». Suppose, that answered correctly 50% interviewees. From this we can hastily conclusion that 50% respondents know the correct answer, but it is not so. If a person does not know the correct answer, then most often chooses it at random. This example shows the need for deep knowledges of probability theory, which will help to make the right conclusions.

The presented work is devoted to the section on the queueing theory, which is engaged in the study of probabilistic processes for the purpose of their management. This knowledge can be useful for a successful modern manager. In order to simplify the application of certain ideas of the queueing system, was give the task of deriving a simple method of approximating recurrent probabilities through geometric progression.

We have a rather significant class of probabilistic processes, which are expressed by recurrent equations, when probability of the current event is determined by the previous probabilities [1]. Such probabilities became the basis of the study.

We can say, that the recurrent equation is a generalization of the famous Fibonacci formula. Therefore, it is desirable not only to give an example of such a connection, and also get formula, similar to Binet's formula, for numbers Fibonacci.

So, from example of throwing a coin, there was an interest in building a method for calculating recurrent possible probabilities of a certain type. Moreover, to derive precise theoretical formulas for their calculations and to offer a successful practical method of their approximation. Method, which would be so simple and obvious, that it could be used by any person, even far from mathematics, and the effectiveness of the method would be useful to specialists in different fields, where mass service issues arise.

Let us consider next task with throwing coin. Let it throw coin, conditionally, hundred times. What is interesting for us is the frequency of the coat of arms appearing twice or more. It's easy search possibility reverse action, that , what for n throws coin never fall out succession twice coats of arms (let's count $n = 100$) [1]. This probabilities form sequence $Q_n = \{q_0, q_1, q_2, q_3, \dots, q_n\}$. Тоді $q_0 = 1, q_1 = 1$, as for zero or one throw can't fall out two coats of arms.

General probability this case, what interesting, equal sum probabilities:

$$q_n = \frac{1}{2}q_{n-1} + \frac{1}{4}q_{n-2}. \quad (1.1)$$

We have a recurrent formula, look like Fibonacci numbers.

We can use similar Binet's formula, which expresses recurrent sequence Φ_n directly trough the number:

$$\Phi_n = \frac{1}{\sqrt{5}} \left(\left(\frac{1+\sqrt{5}}{2} \right)^n - \left(\frac{1-\sqrt{5}}{2} \right)^n \right). \quad (1.2)$$

And get finally needed probability:

$$\begin{aligned}
 q_n &= \frac{\Phi_{n+2}}{2^n} = \frac{\frac{1}{\sqrt{5}} \left(\left(\frac{1+\sqrt{5}}{2} \right)^{n+2} - \left(\frac{1-\sqrt{5}}{2} \right)^{n+2} \right)}{2^n} = \frac{\frac{1}{\sqrt{5}} \left(\left(\frac{1+\sqrt{5}}{2} \right)^2 \left(\frac{1+\sqrt{5}}{2} \right)^n - \left(\frac{1-\sqrt{5}}{2} \right)^2 \left(\frac{1-\sqrt{5}}{2} \right)^n \right)}{2^n} = \\
 &= \frac{\frac{1}{2\sqrt{5}} \left((3+\sqrt{5}) \left(\frac{1+\sqrt{5}}{2} \right)^n - (3-\sqrt{5}) \left(\frac{1-\sqrt{5}}{2} \right)^n \right)}{2^n} = \\
 &= \frac{1}{2\sqrt{5}} \left((3+\sqrt{5}) \left(\frac{1+\sqrt{5}}{4} \right)^n - (3-\sqrt{5}) \left(\frac{1-\sqrt{5}}{4} \right)^n \right)
 \end{aligned} \tag{1.3}$$

Also we can get same expression such effective reasoning. Suppose, what probability is expressed geometric progression with common ratio g , to wit $q_n = g^n$, and try to find such g with using the formula (1.1).

Let's substitute directly $q_n = g^n$ in the specified formula. We will get:

$$q_n = \frac{1}{2}q_{n-1} + \frac{1}{4}q_{n-2} \Rightarrow g^n = \frac{1}{2}g^{n-1} + \frac{1}{4}g^{n-2}$$

Let's reduce two parts this equation on g^{n-2} , we will get quadratic equation relatively which we search g^{n-2} :

$$g^2 = \frac{1}{2}g + \frac{1}{4}, \tag{1.4}$$

which called *characteristic equation*. It solved standard method. We will get two solutions of the equation:

$$g_1 = \frac{1+\sqrt{5}}{4} \quad \text{i} \quad g_2 = \frac{1-\sqrt{5}}{4}, \tag{1.5}$$

which, by virtue of construction, obviously, satisfy the formula (1.1), thanks to formula derived to from it (1.4).

But for build sequence not enough, for it satisfy construction of a formula. Needed to ask Its first two members, in us case it $q_0 = 1, q_1 = 1$.

Found geometrical progressions with common ratios (1.5) this requirement not satisfy. If more precisely, they satisfy condition for $q_0 = 1$. Really, g_1 or g_2 in zero degree are equal $g_1^0 = g_2^0 = 1$, like any numbers, which not equal zero.

But $g_1^1 = \left(\frac{1+\sqrt{5}}{4} \right) \neq q_1 = 1$ and $g_2^1 = \left(\frac{1-\sqrt{5}}{4} \right) \neq 1$. Easy to show direct

substitution, if sequences g_1^n and g_2^n satisfy to formula (1.1), and sequence $a_n = xg_1^n + yg_2^n$, where x and y – довільні arbitrary fixed numbers, also satisfying (1.1). Because we try selection x and y , for to perform

$a_0 = q_0 = 1$ and $a_1 = q_1 = 1$. For it let's solve system equations with two variables:

$$\begin{cases} g_1^1 x + g_2^1 y = 1 \\ g_1^0 x + g_2^0 y = 1 \end{cases} \Rightarrow \begin{cases} \frac{1+\sqrt{5}}{4} x + \frac{1-\sqrt{5}}{4} y = 1 \\ x + y = 1 \end{cases} \Rightarrow \begin{cases} \frac{1+\sqrt{5}}{4} x + \frac{1-\sqrt{5}}{4} (1-x) = 1 \\ y = 1-x \end{cases} \Rightarrow \begin{cases} y = 1-x \\ \frac{2\sqrt{5}}{4} x = 1 - \frac{1-\sqrt{5}}{4} \end{cases} \Rightarrow \begin{cases} y = 1-x \\ x = \frac{3+\sqrt{5}}{2\sqrt{5}} \end{cases} \Rightarrow \begin{cases} y = 1 - \frac{3+\sqrt{5}}{2\sqrt{5}} = \frac{-3+\sqrt{5}}{2\sqrt{5}} \\ x = \frac{3+\sqrt{5}}{2\sqrt{5}} \end{cases}$$

And finally:
$$q_n = \frac{1}{2\sqrt{5}} \left((3+\sqrt{5}) \left(\frac{1+\sqrt{5}}{4} \right)^n - (3-\sqrt{5}) \left(\frac{1-\sqrt{5}}{4} \right)^n \right). \tag{1.6}$$

Searched sequence equal:

$$q_{100} = \frac{1}{2\sqrt{5}} \left((3 + \sqrt{5}) \left(\frac{1+\sqrt{5}}{4} \right)^{100} - (3 - \sqrt{5}) \left(\frac{1-\sqrt{5}}{4} \right)^{100} \right). \quad (1.7)$$

What is better? First off all, it very important for further research, second member in equation can be immediately neglected in approximate calculations, and written:

$$q_{100} \approx \frac{1}{2\sqrt{5}} (3 + \sqrt{5}) \left(\frac{1+\sqrt{5}}{4} \right)^{100} \quad (1.8)$$

For calculations according to the formula (1.8) not necessarily do 100 multiplications, if take into account, what $a^{100} = a^4(a^{32})^3 = a^4 \cdot a^{32} \cdot a^{32} \cdot a^{32}$ needed three multiplications. If consider, what $a^{32} = (((a^2)^2)^2)^2$ calculation to five multiplications, will needed 5+3=8 multiplications instead 100.

In our case $q_{100} \approx 7,31568 \cdot 10^{-10}$, it's very small value.

It may seem, what this task interesting only for theory and not have practical sense. However, this is not the case. Enough to reformulate it to other way.

For example, there is a workshop with a few working machines, which every hour with probability $p = \frac{1}{2}$ («fall out coat of arms») can needed intervention of the adjuster for tooling their work. Naturally, probability absence this situation (all machine working, «fall out Tail») equal $q = 1 - p = \frac{1}{2}$.

Suppose, adjuster can process within two hours no more than one machine. Stopping two machines in a row for two hours will result in a refusal of service. Probability happen this case, For one work shift (8 hours) («8 throws coin») will equal $1 - q_8 = 1 - 0,22 = 0,78$. Knowledge this probability important during to creation service department, we can use two adjusters.

Initial information for this task will be different: probability p breakage will one smaller than the other, number defects, which can repair adjuster, will much more (it's mean, that need consider the probability of the appearance of not two, but, conditionally, say, 5 defects in a row). The proposed model and method can help solved such tasks. So, this recurrent method brought to completely different level the possibilities of researching to real practical tasks.

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ВПЛИВ КІНОІНДУСТРІЇ НА ТУРИСТИЧНУ ДІЯЛЬНІСТЬ

У сучасному світі кінотуризм набуває неабиякої популярності. Цей вид туризму є незвичайним, адже бере свій початок з кінострічки. Побачивши яскраву картинку, глядач захоплюється мальовничими